International Journal of Innovation and Research in Educational Sciences
Volume 2, Issue 1, ISSN (Online): 2349–5219

Dynamics Transmission of Lassa Fever Disease

James. T.O., Abdulrahman. S., Akinyemi. S and Akinwande, N.I
Corresponding Author: tolujam@yahoo.com_192

Abstract: **In this paper, we proposed a Mathematical model of Lassa fever disease dynamics. We developed a model using a set of ordinary differential equations. The equilibrium states were obtained and analyzed for stability. We discovered that the zero equilibrium state is stable when the birth rate of the human population is less than the death rate and same when the birth rate of the mastomysnatalensis (reservoir) is less than the total death rates.**

Keywords: **Equilibrium States. Lassa Fever, Mathematical Model, Stability.**

1. INTRODUCTION

Lassa fever is a zoonotic disease which is transmitted from an infected animal to a human, but may also be transmitted from person-to-person. The animal host of Lassa virus is rat specie known as mastomy (mastomynatalensis in particular). It can also be defined as a viral disease that attacks the liver, nervous system, spleen and kidney, causing them to bleed, hence the haemorrhagic fever. It is an acute viral hemorrhagic fever (VHF) first described in 1969 in the town of Lassa in Borno State, Nigeria in the Yedseram river valley at the south end of Lake Chad [4]. Lassa fever is a disease that occurs often in the dry season because dust particles from dead rats carry this virus are more mobile making it easy to inhale it. It is an infection that is endemic inWest African countries of Liberia, Sierra Leone, Guinea and even Central African Republic and Congo DR. The fever is caused by Lassa virus which belongs to the arena virus family, and classified as group V(-)ss RNA. Approximately 15% 15%-20% of patients hospitalized for Lassa fever die from the illness. However, overall only about 1% of infections with Lassa virus results in death. The fever is therefore transmitted through exposure to infected mastomy (air borne diseases or consuming contaminated food) and through direct contact with the blood, urine, feces or other body secretions of a person infected with Lassa virus, but the virus cannot be spread through casual contact (including skin-to-skin contact without exchange of body fluids) [3]. Person-to-person transmission occurs through direct contact with sick patients in both community and health care settings. Those at greatest risk are persons living in rural areas where Mastomys are found [6].

The symptoms of Lassa fever show up anytime between one and three weeks after infection. According to the CDC, symptoms of Lassa are varied and include fever, abdominal pain, malaise, headache, sore throat, muscle pain, chest pain, nausea, vomiting, diarrhea, facial swelling, protein in the urine, encephalitis, and mucosal bleeding. Once infection occurs, the time it takes symptoms to develop, known as the incubation period is between six to twenty-one days after which an acute illness with multi-organs involvement develops [8].

Studied by [7] on Lassa fever examined the steady states of their model for epidemic and endemic situations. A second model which incorporates the effect of vaccination on a subset of the target population was proposed. Their model shows that in the interim control of the rodents carrying the virus and some isolation policy for infected individuals are the best strategies against the spread of the disease. [5] in his work on Lassa separated the susceptible population into two to get the exposed population, S and the reversed population E then infective population, I, and the rat population, R, which he represented by a logistic model with carrying capacity K. A deterministic model for Lassa fever disease was developed by [2]. In a population with vital dynamics, incorporating standard incidence rate, disease induced death and infection due to humans, reservoirs and aerosol (airborne) transmissions. They obtained the basic reproduction number,R_0 which can be used to control the transmission dynamics of the disease and they established the conditions for local and global stability of the disease-free equilibrium.

2. MODEL DEVELOPMENT

The human population is partitioned into the susceptible class S(t) and the infected class I(t).Then the virus carrier (reservoir) population R(T).These populations are represented by a set of ordinary differential equation. It is assumed that a portion of the susceptible class may move to the infected class as a result of interaction with the infected class or the vector. Also a portion of the infected class may move to the susceptible class by way of treatment.

2.1 Basic Assumptions:

i. Members of the susceptible human population S(t) can move to the infected population I(t) via interaction.

ii. Members of the infected population I(t) can as well move to the susceptible population via treatment.

iii. It is assumed that the new birth of susceptible S(t) are susceptible.

iv. Because of high affinity of the Lassa virus with the placenta, the off-springs of the infected is divided between the S (t) and I(t) in the proportion of $(1-\theta)$ and θ respectively.

v. It is assumed that the virus does not kill the vector i.e. their death can be naturally or accidental.

2.2 The Model Equations:

$$\frac{dS(t)}{dt} = \beta_1 - \mu_1 S - (\alpha_1 I + \alpha_2 R)S + (\gamma + (1-\theta)\beta_1)I \quad (1)$$

$$\frac{dI(t)}{dt} = (\alpha_1 I + \alpha_2 R)S - (\mu_1 + \delta_1 + \gamma)I + \theta\beta_1 I \quad (2)$$

$$\frac{dR(t)}{dt} = (\beta_2 - \mu_2 - \delta_2)R \quad (3)$$

The parameters of the model equation are defined as follows:

β_1 Natural birth for the human population

β_2Natural birth for the vector population

μ_1 Natural death rate for the human population

μ_2 Natural death rate for the vector population

δ_1 Death rate for the human population due to infection

δ_2 Death rate of vector population due to other factors like harvesting, fire,

α_1Contracting rate for the susceptible human population as a result of interaction with infected human population

α_2 Contacting rate for the susceptible human population as a result of interaction with faces and urine infected reservoir population.

θ The proportion of the off-spring of the infected human population which are infected at birth $0 < \theta < 1$.

3. Model Analysis

3.1 Equilibrium States of the Model Analysis:

We now solve the model equations to obtain the equilibrium states. At equilibrium

$$\frac{dS(t)}{dt} = \frac{dI(t)}{dt} = \frac{dR(t)}{dt} = 0 \quad (4)$$

Let

$$S(t) = x, \qquad I(t) = y, \qquad R(t) = z,$$

Then the equations (1) – (3) becomes

$$\beta_1 - \mu_1 x - (\alpha_1 y + \alpha_2 z)x + (\gamma + (1-\theta)\beta_1)y = 0 \quad (5)$$

$$(\alpha_1 y + \alpha_2 z)x - (\mu_1 + \delta_1 + \gamma - \theta\beta_1)y = 0 \quad (6)$$

$$(\beta_2 - \mu_2 - \delta_2)z = 0 \quad (7)$$

From (6) we have

$$z = 0 \quad (8)$$

Substitute z = 0 in (5) and (6) to have

$$\beta_1 - \mu_1 x - (\alpha_1 y)x + (\gamma + (1-\theta)\beta_1)y = 0 \quad (9)$$

$$\alpha_1 xy - (\mu_1 + \delta_1 + \gamma)y + \theta\beta_1 y = 0 \quad (10)$$

$$\alpha_1 xy - (\mu_1 + \delta_1 + \gamma - \theta\beta_1)y = 0 \quad (11)$$

Expand and add equation (9) and (10) to obtain

$$x = \frac{\langle\beta_1 - \mu_1 - \delta_1\rangle y + \beta_1}{\mu_1} \quad (12)$$

Now from equation (11), we have

$$[\alpha_1 x - (\mu_1 + \delta_1 + \gamma - \theta\beta_1)]y = 0 \quad (13)$$

Either$y = 0$ or

$$\alpha_1 x - (\mu_1 + \delta_1 + \gamma - \theta\beta_1) = 0,$$

if y = 0 then substitute y = 0 into equation (12) we obtain $x = \frac{\beta_1}{\mu_1}$. Therefore the disease free equilibrium state of

$$(x, y, z) = (\frac{\beta_1}{\mu_1}, 0, 0) \quad (14)$$

If $y \neq 0$ then $\alpha_1 x - (\mu_1 + \delta_1 + \gamma - \theta\beta_1) = 0$

$$x = \frac{(\mu_1 + \delta_1 + \gamma - \theta\beta_1)}{\alpha_1} \quad (15)$$

Substitute equation (15) into (12)

$$\frac{\mu_1 + \delta_1 + \gamma - \theta\beta_1}{\alpha_1} = \frac{\langle\beta_1 - \mu_1 - \delta_1\rangle y + \beta_1}{\mu_1}$$

By cross multiplying and making " y" the subject of the formulae we obtain:

$$y = \frac{\mu_1\langle\mu_1 + \delta_1 + \gamma - \theta\beta_1\rangle - \alpha_2\beta_1}{\alpha_2\langle\beta_1 - \mu_1 - \delta_1\rangle} \quad (16)$$

The endemic equilibrium state

$$(x, y, z = \left(\frac{(\mu_1 + \delta_1 + \gamma - \theta\beta_1)}{\alpha_1}, \frac{\mu_1\langle\mu_1 + \delta_1 + \gamma - \theta\beta_1\rangle - \alpha_1\beta_1}{\alpha_1\langle\beta_1 - \mu_1 - \delta_1\rangle}, 0\right) \quad (17)$$

3.2 Characteristic Equation:

To obtain the characteristic equation of the model equation, we obtain first the Jacobian matrix J, which is defined as presented by Benyah (2008)

$$J = \begin{bmatrix} -\mu_1 - \alpha_1 y & -\alpha_1 x + \gamma + \langle 1-\theta\rangle\beta_1 & -\alpha_2 x \\ \alpha_1 y & \alpha_1 x - \langle\mu_1 + \delta_1 + \gamma - \theta\beta_1\rangle & \alpha_2 x \\ 0 & 0 & \beta_2 - \mu_2 - \delta_2 \end{bmatrix}$$

The characteristic equation of the model is defined by

$$|J - \lambda I| = 0$$

Where λ is the Eigen value and I is an identity matrix of 3 × 3

$$|J - \lambda I| = \begin{vmatrix} -\mu_1 - \alpha_1 y - \lambda & -\alpha_1 x + \gamma + \langle 1-\theta\rangle\beta_1 & -\alpha_2 x \\ \alpha_1 y & \alpha_1 x - \langle\mu_1 + \delta_1 + \gamma - \theta\beta_1\rangle - \lambda & \alpha_2 x \\ 0 & 0 & \beta_2 - \mu_2 - \delta_2 - \lambda \end{vmatrix} = 0$$

$$\langle\beta_2 - \mu_2 - \delta_2 - \lambda\rangle \begin{vmatrix} -\mu_1 - \alpha_1 y - \lambda & -\alpha_1 x + \gamma + \langle 1-\theta\rangle\beta_1 \\ \alpha_1 y & \alpha_1 x - \langle\mu_1 + \delta_1 + \gamma - \theta\beta_1\rangle - \lambda \end{vmatrix} = 0$$

$$\langle\beta_2 - \mu_2 - \delta_2 - \lambda\rangle = 0$$

Or

$$\langle\mu_1 + \alpha_1 y + \lambda\rangle\langle\mu_1 + \delta_1 + \gamma - \theta\beta_1 - \alpha_1 x + \lambda\rangle + \langle\alpha_1^2 xy - \alpha_1\gamma y - \alpha_1\beta_1 y + \theta\alpha_1\beta_1 y\rangle = 0$$

$\langle\mu_1+\lambda\rangle\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\lambda\rangle+\alpha_1y\,\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\lambda\rangle+\langle\alpha_1^2xy-\alpha_1\gamma y-\alpha_1\beta_1y+\theta\alpha_1\beta_1\,y\rangle=0$

$\langle\mu_1+\lambda\rangle\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\lambda\rangle+\alpha_1y\,\langle\mu_1+\delta_1-\beta_1+\lambda\rangle=0$

$\langle\mu_1+\lambda\rangle\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\lambda\rangle+\alpha_1y\,\langle\mu_1+\lambda\rangle+(\alpha_1y\,\langle\delta_1-\beta_1\rangle)=0$

$\langle\mu_1+\lambda\rangle\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y+\lambda\rangle+(\alpha_1y\,\langle\delta_1-\beta_1\rangle)=0$

And so the characteristics equation is given by

$\langle\beta_2-\mu_2-\delta_2-\lambda\rangle\langle\mu_1+\lambda\rangle\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y+\lambda\rangle+(\alpha_1y\,\langle\delta_1-\beta_1\rangle)=0$ (18)

3.3 Stability Analysis of the Disease Free Equilibrium States:

Having established the equilibrium states. We now investigate the stability analysis of the zero equilibrium states. To obtain this**,** from equation (14) we obtained the zero equilibrium state

i.e. (x, y, z) = $(\frac{\beta_1}{\mu_1}, 0, 0)$

and the characteristics equation for the disease free equilibrium state is obtained by substituting equation (14) into (18) to obtain.

$\langle\beta_2-\mu_2-\delta_2-\lambda\rangle\langle\mu_1+\lambda\rangle\,\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1\frac{\beta_1}{\mu_1}+\lambda\rangle=0$ (19)

Either

$$\left(\beta_2-\mu_2-\delta_2-\lambda\right)=0 \quad (20)$$

$$\langle\mu_1+\lambda\rangle=0 \quad (21)$$

$$\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1\frac{\beta_1}{\mu_1}+\lambda\rangle \quad (22)$$

Now Considering (20)

$\lambda_1=\beta_2-\mu_2-\delta_2\;\lambda_1<0$

If $\lambda_1=\beta_2-\mu_2-\delta_2<0$
i.e.$\beta_2<\mu_2+\delta_2$

From (21)
$\lambda_2=-\mu_1$

1) If $\lambda_2<0$ *then* $\lambda_2=-\mu_1<0$

From(22)

$\lambda=\langle\theta\beta_1+\alpha_1\frac{\beta_1}{\mu_1}-\mu_1-\delta_1-\gamma\rangle<0$

$\langle\theta\beta_1+\alpha_1\frac{\beta_1}{\mu_1}\rangle<\mu_1+\delta_1+\gamma$

$\beta_1\langle\theta\mu_1+\alpha_1\rangle<\mu_1(\mu_1+\delta_1+\gamma)$

is stable and unstable if

$\beta_1\langle\theta\mu_1+\alpha_1\rangle>\mu_1(\mu_1+\delta_1+\gamma)$

3.4 Stability Analysis of Endemic Equilibrium State:

Recall that at endemic equilibrium state, (x, y, z) is given by equation (17) respectively. Here the vector is absent, but the vector is introduced into the human population. This is because parts of the human population already have the virus; more so, (z = 0) does not mean absence of the virus but rather a Mathematics requirement. Substituting (17) in to (18), we have

Either

$$\beta_2-\mu_2-\delta_2-\lambda=0$$

Or

$\langle\mu_1+\lambda\rangle\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y+\lambda\rangle+(\alpha_1y\,\langle\delta_1-\beta_1\rangle)=0$ (23)

Further expansion of (23) gives

$\lambda^2+\lambda\,\langle2\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y\rangle+\mu_1\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y\rangle+(\alpha_1y\,\langle\delta_1-\beta_1\rangle=0$

Equation (23) is of the form

$$\lambda^2+\langle\mu_1+\omega\rangle\lambda+(\mu_1\omega+\rho)=0 \quad (24)$$

Where ω,ρ are given by

$$\omega=\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y \quad (25)$$

$$\rho=(\alpha_1y\,\langle\delta_1-\beta_1\rangle \quad (26)$$

With the variables x *and* y, given by the equation (15) and (16) respectively as parameters of the model equations.

The solution to (24) is

$$\lambda=\frac{-(\mu_1+\omega)\pm\sqrt{(\mu_1+\omega)^2-4(\mu_1\omega+\rho)}}{2} \quad (27)$$

The sufficient condition for the stability of the endemic equilibrium state is that the Eigen values of (27) are negative (i.e. $\lambda<0$), otherwise unstable.

For $\lambda<0$

hence $\lambda<0$

$$(\mu_1+\omega)>\sqrt{(\mu_1+\omega)^2-4(\mu_1\omega+\rho)}$$

Squaring both sides to have

$$\left(\mu_1+\omega\right)^2 > \left(\mu_1+\omega\right)^2 - 4\left(\mu_1\omega+\rho\right)\ \mu_1\omega \succ -\rho \quad (28)$$

Substituting (25) and (26) into (28) to obtain

$$\mu_1\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x+\alpha_1y\rangle > -\alpha_1y\,\langle\delta_1-\beta_1\rangle$$

$$\mu_1\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x\rangle > -\alpha_1y\,\langle\delta_1-\beta_1+\mu_1\rangle$$

$$\mu_1\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1x\rangle > \alpha_1y\,\langle\beta_1-\mu_1-\delta_1\rangle \quad (29)$$

Substitute (17) into (29) to have

$$\mu_1\,\langle\mu_1+\delta_1+\gamma-\theta\beta_1-\alpha_1\,\langle\frac{(\mu_1+\delta_1+\gamma-\theta\beta_1)}{\alpha_1}\rangle\rangle$$
$$> \alpha_1\,\langle\frac{\mu_1\langle\mu_1+\delta_1+\gamma-\theta\beta_1\rangle-\alpha_1\beta_1}{\alpha_1\langle\beta_1-\mu_1-\delta_1\rangle}\rangle\,\langle\beta_1-\mu_1-\delta_1\rangle$$

$$0>\mu_1\langle\mu_1+\delta_1+\gamma-\theta\beta_1\rangle-\alpha_1\beta_1$$

$$\mu_1\theta\beta_1+\alpha_1\beta_1>\mu_1\langle\mu_1+\delta_1+\gamma\rangle$$

$$\beta_1\langle\mu_1\theta+\alpha_1\rangle > \langle\mu_1+\delta_1+\gamma\rangle \quad (30)$$

(30) is stable, otherwise unstable

4. Discussion

The disease free equilibrium state (is a state of complete eradication of Lassa fever) will be stable if$\frac{\beta_1\langle\mu_1\theta+\alpha_1\rangle}{\mu_1} < \langle\mu_1+\delta_1+\gamma\rangle$where $\frac{\beta_1\langle\mu_1\theta+\alpha_1\rangle}{\mu_1}$is the number of latent infections produced and $\langle\mu_1+\delta_1+\gamma\rangle$is the total removal rate from the infectious class .We also observed that from (3.4) the endemic state is stable if $\frac{\beta_1\langle\mu_1\theta+\alpha_1\rangle}{\mu_1} > \langle\mu_1+\delta_1+\gamma\rangle$which implies that the treatment of Lassa fever could lead to the eradication of the disease hence the population is sustain.

5.Conclusion

In this paper, we presented a dynamics transmission of Lassa fever disease. The result of the model analysis shows that the zero equilibrium state of the model equation will be stable when the birth rate of the human population is less than the death rate i.e. $\beta_1 < \mu_1$ and same when the birth rate of the vector is less than the total death rate i.e. $\beta_2 < \mu_2+\delta_2$. Further analysis of equation (23); $\lambda^2+a\lambda+b=0$ shows that the non-zero equilibrium state is unstable since $a > 0, while\ b > 0$. This confirms the present situation where it is not practicable to control the vector in the prevalence West African states.

References

[1] F.Benyah, "Introduction to epidemiological modeling". 10th Regional College On Modelling, Simulation and Optimization"./University of Cape Coast. 2008
[2] M. Bawa, S.Abdulrahman, O.R Jimoh, and N.U.Adebara,. "Stability Analysis of the Disease –free equilibrium State for Lassa fever Disease". Journal of Science, Technology, Mathematics and Education (JOSTMED).Vol 9(2), 2013, pp 115-123.
[3] Centers for Disease Control and Prevention. Lassa fever fact sheet. 2013
[4] J.D.Frame, J.M. Baldwin, D.J.Gocke, J.M. Troup Lassa fever, a new virus disease of man from West Africa. Am. J. Trop. Med. Hyg.;1970, vol19 pp670–676.
[5] Inyama, "A Mathematical model for Lassa fever with reserved population " 2009, Unpublished.
[6] R.A. Keenlyside, J.B. McCormick, K. M. Johnson. "Case- control study of Mastomys Natalensis and Human in Lassa virus – infected Household in Sierra Leone". AMJ. Trop. Med. Hygiene, 1983 pp839.
[7] D. Okuonghae and I. Okuonghae A mathematical model for Lassa fever Journal of National Association of Mathematical Physics, 2006, vol.10, pp 457- 464.
[8] World Healths Organization, WHO Lassa fever sheet 2000, No: 179 Geneva