International Journal of Innovation and Research in Educational Sciences
Volume 6, Issue 2, ISSN (Online) : 2349–5219

A Case Teaching Study of Mathematics Expectation and Variance

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Date of publication (dd/mm/yyyy): 04/04/2019

***Abstract* – Mathematical expectation and variance play an important role in economic decision-making. Based on the example of investor selection and the concept of mathematical expectation and variance, this paper analyzes their application methods and steps in economic decision-making and introduces this case into classroom teaching. The results show that mathematical expectation and variance can provide practical methods for economic decision-making, and the case of economic decision makes classroom teaching more practical.**

***Keywords* – Mathematical Expectations, Variance, Economic Decision-Making, Investor's Choice, Case Teaching.**

I. INTRODUCTION

A. *Introduction to Mathematical Expectations and Variance*

1.1.1 *Definition and Annotation of Mathematical Expectations*

Definition 1 (mathematical expectation of discrete random variables): Let X be a discrete random variable and its probability distribution law be $P(X=x_i)=p_i, i=1,2,3,\cdots$, if the series converges absolutely, mathematical expectation of X exists, and this series is called the mathematical expectation of the random variable X (abbreviated as expectation or mean), denoted as $E(X)$, That is $E(X)=\sum_{i=1}^{\infty} x_i p_i$.

Notes:

I. The mathematical expectation is the probability average, which depicts the central position of all possible values of the random variable [1].

II. If the series absolutely converges, the mathematical expectation is said to exist, otherwise the expectation does not exist.

Definition 2 (Mathematical Expectation of Continuous Random Variables):

Let X be a continuous random variable with a probability density $f(x)$. If the integral $\int_{-\infty}^{\infty} xf(x)\,dx$ converges absolutely, then the mathematical expectation of X exists, and this product is called the mathematical expectation of the random variable X, denoted as $E(X)$, i.e., $E(X)=\int_{-\infty}^{\infty} xf(x)dx$.

Notes:

If the integral absolutely converges, then the mathematical expectation of X exists, otherwise the expectation does not exist [2].

1.1.2 *Definition and Calculation of Variance*

Define 3(Variance of Random Variables):

Let X be a random variable. If $E\left\{\left[X-E(X)\right]^2\right\}$ exists, it is called the variance of X, denoted as $D(X)$ or $Var(X)$, IE $D(X)=E\left\{\left[X-E(X)\right]^2\right\}$.

Calculation of Variance:

I. If X is a discrete random variable and its distribution law is $P(X=x_i)=p_i, i=1,2,3,\cdots$, then $D(X)=\sum_{i=1}^{+\infty}[x_i-E(X)]^2 p_i$.

II. If X is a continuous random variable with a probability density of $f(x)$, then $D(X)=\int_{-\infty}^{+\infty}[x-E(X)]^2 f(x)dx$.

III. The following formula can be derived from the definition: $D(X)=E(X^2)-E^2(X)$.

B. *Brief Introduction of Case Teaching*

1.2.1 *Origin of Case Teaching*

Case teaching originated in 1920 and was advocated by Harvard Business School [3]. At that time, it was teaching in a very unique case form. These cases came from the real situation or event of business management. Through this way, it helps to cultivate and develop students to actively participate in classroom discussions. After implementation, it is quite successful.

1.2.2 *Case Teaching Requirements*

The case should be typical for teaching objectives and should be directly related to the corresponding theoretical knowledge. At the same time, it must be deeply investigated and studied and derived from practice. It must not be made by teachers 'subjective speculation and fiction. Reality is a prerequisite, but cases cannot be a list of examples and data. Teachers should get rid of the boring way of writing textbooks and mobilize some literary techniques. The case is better to have fierce contradictions and conflicts. There are no obvious solutions and conclusions. It is necessary to encourage students to find ways to make decisions and handle them, and different methods will produce different results. Assuming that you can wear it at a glance, or that there is only one good and one bad ending, such a case will not cause controversy and the student will lose interest [4]. In this sense, the more complex and diverse the outcome of a case, the more valuable it is.

1.2.3 *Advantages of Case Teaching*

Case teaching can mobilize students 'learning initiative. In teaching, due to constantly changing the teaching form, the student's brain is excited and his attention can be adjusted in time, which is conducive to maintaining the best state of the student's spirit. The greatest feature of case teaching is its authenticity. Because the content of teaching is a concrete example, coupled with the use of images, intuition, and vivid forms, it gives people a sense of being in their own situation and is easy to learn and understand. Case studies can brainstorm. Teachers discuss and think with everyone in the classroom. Students are not busy taking notes in the classroom, but discuss problems together. Due to the mobilization of collective wisdom and strength, it is easy to open up ideas and receive good results.

II. Examples and Case Teaching of Expectation and Variance in Economic Decision Making

A. *Question Raised*

Investors want to buy wealth management products for $10,0000, and they tend to buy Higher-Yielding wealth management products in order to add value. The staff recommended three products A, B, and C, gave a profit and loss analysis of the three financial products, and also mapped out the expected income X, Y, Z, of the three products one year later, as shown in tables 1, 2, and 3 below:

Table 1 Expected income and corresponding probability of A (in $10,000)

Expected income X	5	4	2	1	0	-1	-2
corresponding probability P	0.2	0.05	0.2	0.15	0.2	0.05	0.15

Table 2. Expected income and corresponding probability of B (in $10,000).

Expected income Y	5	3	2	1	0	-1	-2
corresponding probability P	0.15	0.1	0.15	0.3	0.05	0.2	0.05

Table 3 Expected income and corresponding probability of C (in $10,000).

Expected income Y	4	3	2	1	0	-1	-2
corresponding probability P	0.05	0.1	0.3	0.15	0.3	0.05	0.05

After seeing the above picture, the investor is confused and does not know how to choose. Then let the student group discuss whether the knowledge of expectations and variance can help him make reasonable choices.

B. *Problem Analysis*

The primary problem for teachers to guide students to help investors make choices is the need to clarify the selection criteria for financial products. Investors choose to buy wealth management products because of their high returns relative to other financial methods [5]. Therefore, high returns can be used as a selection criterion for purchasing wealth management products. But given the popular phrase "high yield, high risk" in wealth management markets, investors also want to minimize risk while guaranteeing certain returns. Therefore, another selection criterion needs to be considered: low risk.

Having identified the two selection criteria of high yield and low risk, investors are faced with another problem. Since the earnings of wealth management products are all uncertain, how can they quantify the expected returns after determining the selection criteria? The following teachers guide students to use high income and low risk as the selection criteria to quantify the expected benefits of each financial product to help investors make choices.

C. *Problem Solving*

2.3.1 *Select Standard I High Earnings*

Since there are several values of wealth management product earnings, only the probability distribution of expected earnings cannot be accurately known for each product after one year, so mathematical expectations can

be used to represent this expected income [6]. At this point, teachers can lead to the concept of mathematical expectations.

According to the probability distribution data of the expected income of wealth management products A, B, and C, using the definition of mathematical expectation 1, students can calculate it themselves

$$E(X) = 5\times 0.2 + 4\times 0.05 + 2\times 0.2 + 1\times 0.15 + 0\times 0.2 + (-1)\times 0.05 + (-2)\times 0.15 = 1.4,$$
$$E(Y) = 5\times 0.15 + 3\times 0.1 + 2\times 0.15 + 1\times 0.3 + 0\times 0.05 + (-1)\times 0.2 + (-2)\times 0.05 = 1.35,$$
$$E(Z) = 4\times 0.05 + 3\times 0.1 + 2\times 0.3 + 1\times 0.15 + 0\times 0.3 + (-1)\times 0.05 + (-2)\times 0.05 = 1.1.$$

Teachers can show the mathematical expectations of calculating, using the SUMPRODUCT function in Excel software, as shown in table 4:

Table 4 Mathematical expectations of expected earnings (in $10,000)

product	Expectations of expected benefits
A	EX = 1.4
B	EY = 1.35
C	EZ = 1.1

Using Excel field demonstration calculation can not only save time for traditional classroom blackboard writing demonstrations, but also make students feel the powerful role of mathematical software in learning. It is easy to see from the table that if the choice is based on the maximum expected return, investors should choose the product A with the highest average return. However, in this kind of venture investment, high returns are often accompanied by high risks. Venture investors invest in the investment and must pay attention to whether the risk of the invested project is within his acceptable range. If the risk is more than he can handle, even if the expected yield is high, investors will probably not take the risk to invest. Therefore, the following teachers lead students to consider low risk.

2.3.2 *Selection Standard II Low Risk*

In a certain environment, at a certain time, the measure of income uncertainty in economics can be measured by variance. Variance is the quantity used to measure the degree of deviation between the value of a random variable and its mathematical expectation. The smaller the variance, the smaller the deviation between the random variable and its mathematical expectation, and the more the value of the random variable is concentrated near the expected value [7]. The concept of variance is closely related to life, and it is widely used in risk decision-making, economic decision-making, securities investment, benefit assessment, product quality assessment and so on. The teacher can lead to the definition of variance and the calculation formula at this time.

Using variance to help investors choose according to low risk, we can calculate based on the probability distribution table of the expected income of the three financial management products.

$$E(X^2) = 5^2\times 0.1 + 4^2\times 0.15 + 2^2\times 0.2 + 1^2\times 0.15 + 0^2\times 0.2 + (-1)^2\times 0.05 + (-2)^2\times 0.15 = 7.4,$$
$$E(Y^2) = 5^2\times 0.15 + 3^2\times 0.1 + 2^2\times 0.15 + 1^2\times 0.3 + 0^2\times 0.05 + (-1)^2\times 0.2 + (-2)^2\times 0.05 = 5.95,$$
$$E(Z^2) = 4^2\times 0.05 + 3^2\times 0.1 + 2^2\times 0.3 + 1^2\times 0.15 + 0^2\times 0.3 + (-1)^2\times 0.05 + (-2)^2\times 0.05 = 3.3.$$

Then by the formula of variance,

$D(X)=7.4-1.4^2=5.44,\ D(Y)=5.95-1.35^2=4.1275,\ D(X)=3.3-1.1^2=2.09,$

The following table 5 can also be calculated from Excel software:

Table 5. Variance of expected benefits (in $10,000).

product	Variance of expected benefits
A	DX = 5.44
B	DY = 4.1275
C	DZ = 2.09

As can be seen from Table 5, if you use low risk as the selection criterion for financial products, you obviously want to choose product C, and if you obviously choose product A as the selection criterion for high income, then what is the choice? The teacher leaves some time for discussion and reflection, and then suggests that in order to make the investment more secure, the following two options are considered for decision-making.

III. Comprehensive Decision and Conclusion

In order to select products with high yield and low risk, it is advisable to assess the risk in terms of product C's income and risk.

Table 6. Risk Assessment form (in $10,000).

	A		B		C
		Comparison with C		Comparison with C	
income	1.4	Increase 0.3	1.35	Increase 0.25	1.1
risk	5.44	Increase 3.35	4.1275	Increase 2.0375	2.09

As can be seen from table 6, product A gains increased by 0.3 and risk increased by 3.35 compared to product C. Compared with C, product B has an increase of 0.25 and a risk increase of 2.0375. The increase in income is higher than the increase in risk. Taking into account, investors can be advised to choose product B. If students raise objections here, they can be discussed in groups.

The teacher finally proposed that if more objects are selected and the amount of data is greater, the above selection process can be further quantified. Dividing the increase in earnings from the increase in risk in the above table to obtain a critical figure, table 6 can be changed to table 7 as follows:

Table 7. Risk Assessment form (in $10,000).

	A		B		C
		Comparison with C		Comparison with C	
income	1.4	Increase 0.3	1.35	Increase 0.25	1.1
risk	5.44	Increase 3.35	4.1275	Increase 2.0375	2.09
Increase in earnings/increase in risk		0.0896		0.1227	

Judging by the ratio, product B has a clear advantage, so investors are recommended to choose product B. At this point, the teacher can tell the student that the data will talk, and the data allows us to choose product B. If students still have different ideas, they can be advised to continue the discussion after class.

The teacher concludes later as follows: Using mathematical expectations and variance to make comprehensive decisions, we can first estimate a relatively average return value based on mathematical expectations, and provide a specific data basis for decision-making, although it is not completely accurate. But it can provide a concrete direction for decision-making. Variance can reflect the fluctuation level of data and help people to make relatively accurate judgments and correct choices. Taken together, the steps for applying mathematical expectations and variance to comprehensive decision-making are as follows:

I. Introduction of random variables to express uncertainty issues;
II. Applying mathematical expectations to achieve the desired goals;
III. Application of variance to assess the risk of achieving the desired objectives;
IV. Comprehensive considerations and sound decision-making [8].

In everyday life and economic activities, such as personal consumption, job search, investment, business marketing programmers, etc., decisions on the various situations encountered and possibilities are often required in order to act in the most reasonable manner. Since decision-making is often affected by random factors, decision-making often has certain risks. In order to avoid risk as much as possible, people often take mathematical expectation and variance as the important basis for comprehensive decision-making [9]. The economic decision cases presented in this paper can not only enable students to learn the concept of mathematical expectations and variance, but also enable students to understand the application of mathematical expectations and variance in economic decision-making. When the decision problem is more complex and the data volume is large, the comprehensive decision method of mathematical expectation and variance can provide the decision makers with more powerful reference tools and obtain greater economic and social benefits.

ACKNOWLEDGMENT

The work is supported by Tai'an Science and Technology Development Project: Probability Theory and Applied Research of Mathematical Statistics and Practice of Innovative Teaching Reform (2018ZC0287) and Project of Natural Science Foundation of Shandong province (ZR2016AM06).

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