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The Analysis of Causes and Countermeasures Research on the Failure of Senior High School Students' Triangle Problem Solving

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Abstract **– Solving triangles is an important part of mathematics for senior high school students, which is conducive to cultivating students' problem-solving ability, exercising their mathematical thinking quality and strengthening their awareness of mathematical application. However many high school students have a lot of problems in solving triangles, which leads to problems such as learning difficulties nowadays. This article will analyze the specific causes of high school students' failure to solve the triangle through some phenomena and give corresponding counter measures. In order to promote students' learning of future knowledge, cultivate personal good learning style and quality.**

Keywords **– Solution Triangle, Cause of Failure, Countermeasure.**

I. Introduction

Trigonometry is based on the study of the relationship between the sides and angles of a triangle. Its purpose is to be applied to measurement. It is also a subject that studies the nature and application of trigonometric functions. Trigonometry originated from the quantitative investigation of the relationship between triangles and corners in life practice. It began with the measurement of astronomy by Hipparchus, Menelaus, and Ptolemy in ancient Greece. From the 13th century, the triangular knowledge contained in astronomy was introduced into Europe, and new developments appeared in Europe. Modern trigonometry started with Euler's "Introduction to Infinite Analysis". He defined the unit circle and defined the trigonometric function by the ratio of the function line to the radius. He also used lowercase Latin letters a, b, c to represent the three sides of the triangle, and uppercase Latin letters A, B, C to represent the three corners of the triangle, thus simplify triangle formula [1]. Trigonometry further converted from the research triangle trigonometric solution for the research and its application to become a more complete mathematical branches. In the 16th century, French mathematician Wei Da further systematically trigonometry. In his first book on the study of trigonometry, "Mathematical Principles Applied to Triangles", there are detailed descriptions of solving right triangles and oblique triangles. In addition, there are tangent theorem and difference product theorem, etc. In the 18th century, the Swiss mathematician Euler first studied trigonometric functions. Euler uses rectangular coordinates to define trigonometric functions, introduces the radian system, solves the problem of trigonometric functions in four quadrants, and adds new vitality to trigonometry [2].

In 1631, trigonometry was introduced to China. In the same year, German missionaries Yuhan Deng, Ruowang Tang , and Ming Dynasty scholar Guangqi Xu compiled the book "The Great Test". "Triangle" was first seen in the "Triangle Algorithm" co-authored by Fengzuo Xue and Mooney Court in 1653. The term "triangle" refers to "trigonometry", "trigonometry" or "trigonometry". In fact, until the compilation and publication committee of the Chinese academy of sciences compiled the "Mathematical Nouns" in 1956, the three meanings were still the same, traditional trigonometry is based on the study of the relationship between the

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angles of plane triangles and spherical triangles to achieve the purpose of measurement [3]. Chinese middle school mathematics courses now include plane triangles and spherical geometry. People's education press a edition the second edition of February 2007, "Comprehensive mathematics compulsory 4 for general high school curriculum standard experimental textbook" and "Compulsory mathematics compulsory 5 for general high school curriculum standard experimental textbook" include trigonometric functions and trigonometry. From the point of view of the layout of teaching materials, the unit circle is used to study the definition, image and nature of trigonometric functions, focusing on cultivating students' thinking of combining numbers and shapes, classification and discussion, and equations [4]. More emphasis on the practical value of trigonometric functions, creating powerful conditions for students to discover knowledge themselves.

Solving triangles makes the solution of many geometric problems quantitative. Use the formula to express the quantitative relationship between the triangle sides and corners or sides and areas. Then simplification, can be solved or that some of the geometry problems, thus avoiding the many complex auxiliary lines. The triangle solution is also widely used in physics, engineering, technology and other fields. The compulsory five-solution triangle is a continuation of the right-angle main angle of the junior high school. Students can re-understand the triangle from a quantitative perspective, making the triangle an important tool for studying geometric problems, and it is the intersection of many high school mathematics [5]. According to the current situation of the survey, although the students have mastered the basic knowledge, there are still many obstacles in solving the problem of the sides and angles of the triangle. This article summarizes the causes of failure and gives corresponding solutions.

II. THE CURRENT SITUATION OF SENIOR HIGH SCHOOL STUDENTS LEARNING TO SOLVE TRIANGLES

Aiming at the current situation of senior high school students' learning, a questionnaire survey was conducted on the senior high school students of their own tutoring about the solution of triangle learning. The content of the survey is shown in Table II-1.

Table II-1. Questionnaire for senior high school students to solve triangle learning.

Survey Content	Question Number Involved
Investigation of the senior high school students' knowledge of the triangle solution	1,2
Survey of senior high school students' views on triangle learning	3
Survey on the application of basic knowledge of triangle solution by senior high school students	4,5,6
Investigation on difficulties of senior high school students in learning triangle	7,8
Investigation on problems of triangle solution of senior high school students and the reasons for failure	9,10

According to the survey results of question 1 and 2: 34% of the high school students have mastered the knowledge of solving triangles, 21% of the students have not mastered the knowledge of solving triangles, and 45% of the students have mastered the knowledge of solving triangles. According to the question 8: the main reasons for the difficulty in solving triangles in high school students are that they do not fully grasp the basic knowledge, cannot apply the formulas they have learned, ignore the situation of multiple solutions, and lack of problem-solving skills. The proportions are respectively: 26%, 39%, 21% and 14%.

Found the Following Phenomenon:

Although students master the knowledge that will be used in the process of solving triangles, they have difficulties in applying the theorem of sine and cosine.

It is easy for students to ignore the phenomenon that there are multiple situations in the answer to a question in the triangle.

It is difficult for students to use triangle formulas flexibly, which shows that it is easy to make mistakes when solving triangle identity transformation problems.

It is easy for students to ignore the content of the plane vector involved in solving triangles as required by the previous knowledge.

III. Analysis of the Causes of Senior High School Students' Failure to Solve the Triangle

A. *Problems and Analysis of Senior High School Students' Practice in Classroom*

Problems Embodied in Specific Exercises:

Randomly select two classes (110 students in total) of Grade 1 of Dehui city experimental high school in Jilin province for class exercise test (difficulty coefficient: 0.63, discrimination: 0.47). A total of 110 test papers were distributed, and 107 test papers were recovered. The recovery rate was 97.3%, the effective volume was 105 copies, and the effective rate was 98.1%. Found some problems, as follows:

The first exercise: (difficulty coefficient: 0.55, discrimination: 0.42) in triangle ABC, ∠A = 60 °, AB = 6, AC = 2, D is the midpoint of BC.

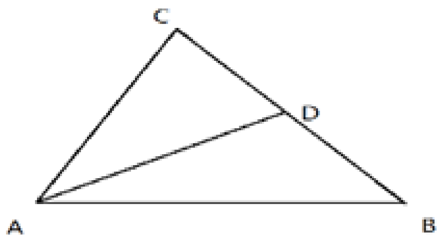

Fig. III-1.

(1) Find the length of the BC side.

(2) Find the length of AD.

85% of students can find the length of the BC side through the cosine theorem, namely:

$$BC^2 = AB^2 + AC^2 - 2|\overline{AB}||\overline{AC}|\cos A = 28$$
$$BC = 2\sqrt{7}$$

13% of the students did not have knowledge of the cosine theorem, so they did not solve the length of BC.

In the second question, there are four ways to find the AD length. The main method used by 31.8% of the students who made the result is to use the cosine theorem and its deformation. The process is as follows:

Using the BC length found in the first question, the cosine of ∠C can be found.

$\cos C = \frac{AC^2 + BC^2 - AB^2}{2|\overrightarrow{AC}||\overrightarrow{BC}|} = -\frac{\sqrt{7}}{14}$, Use the cosine theorem to find AD: $AD^2 = AC^2 + CD^2 - 2|\overrightarrow{AC}||\overrightarrow{CD}|\cos C = 13, AD = \sqrt{13}$

The second method is to use vector to solve: according to the vector addition rule, and then square the vector. As shown in Figure III-2 (also a derivation method of the cosine theorem).

The length of AD can be obtained by $\overrightarrow{AD}^2 = (\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC})^2$ expansion calculation.

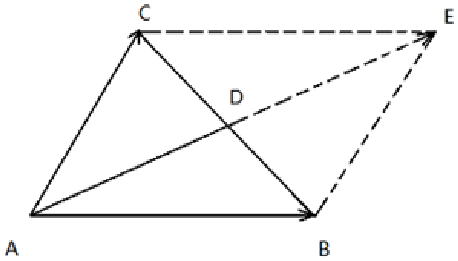

Fig. III-2.

The third method : Use A as the origin and AB as the x-axis to establish a plane rectangular coordinate system, as shown in figure III-3. You can write the coordinates of point B (6, 0), the coordinates of point C $(1,\sqrt{3})$, point D is the midpoint of point B and point C, so the coordinate of D is $(\frac{7}{2}, \frac{\sqrt{3}}{2})$ then the length of AD is from point D to origin distance is $\sqrt{13}$ [6].

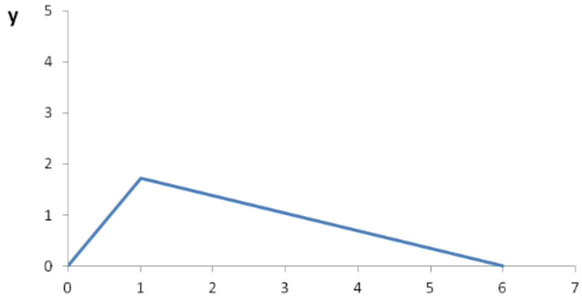

Fig. III-3.

Through the analysis of the test results: students can find the first question but not the second question. This shows that students still understand the basic application of the cosine theorem. That is, the application of the cosine theorem itself, the third side is obtained by knowing the size of the diagonal angle between the two sides of the triangle and the other side. However, the understanding of the cosine theorem is not thorough, and the application is not yet skilled and flexible. Students will use the first two methods to solve, only 6.67% of students use method three to solve. It shows that students generally do not master the method of using plane vector and establishing rectangular coordinate system to solve triangle.

The second exercise: (difficulty coefficient: 0.63, discrimination: 0.48) in triangle ABC, a = $\sqrt{5}$, b = $\sqrt{15}$, $\angle$A = 30 °, then B is equal to ______ this topic involves very few knowledge points, only the sine theorem. However, when faced with this problem, 39% of students often ignore that the angle range of $[0, \pi]$, the triangle is that there are two corresponding angles when the sine value is greater than 0 and less than 1. So write the following steps: $\frac{a}{\sin A} = \frac{b}{\sin B}, \frac{\sqrt{5}}{\sin 30°} = \frac{\sqrt{15}}{\sin B}$

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$\sin B = \frac{\sqrt{3}}{2}, B = \frac{\pi}{3}$. This ignores the result of $B = \frac{2\pi}{3}$.

In addition to the above typical problems, the following problems were discovered during the students' specific problem solving process: calculation errors or knowledge points discovered by calculation errors were not understood clearly. For example, the cosine value of a certain angle of the triangle is calculated as a value greater than 1, and the sine value of one angle of the triangle is calculated as a negative number. The result obtained is not within the scope, but also shows that the student did not check the result from another aspect [7]. The question was not carefully reviewed, and the known conditions in the question were misunderstood.

B. *Questionnaire Survey Results and Analysis*

The questionnaire consists of 16 questions. The design of the questions is mainly based on the communication with senior high school mathematics teachers and personal experience to list some factors that may affect the problem solving difficulties of most high school students, such as the lack of solving methods, the lack of basic knowledge, the fear of solving problems, etc. The causes of the problem solving difficulties of senior high school students are mainly investigated from multiple angles. The detailed distribution of students' questionnaires is shown in the following table III-1:

Table III-1. Distribution of specific content of the student questionnaire survey.

Survey Content	Question Number Involved
A survey of senior high school students' math scores	1
Survey of senior high school students' views on solving mathematics problems	2
Survey of senior high school students' difficulty in solving triangle problem and its causes	3,4
Investigation of difficult problem solving for students due to inadequate basic knowledge	5,6,7
The investigation of students' failure to solve problems due to arithmetic problems	8
Investigation on the thinking state of senior high school students in solving math problems	9
Investigation of difficult problem solving for students due to missing problem solving method	10,11
Investigation on the mental state of senior high school students in solving math problems	12,13
A survey of senior high school students' habit of solving mathematical problems	14,15
Investigation of mathematics problem solving standards in senior high school life	16

The results of this survey are organized as follows: First of all, the senior high school students' math scores were investigated, and the full score of the test was 150 points. Of the 105 students, 31 students scored above 120 points and 23 students failed. The survey results are shown in table III-2 and figure III-5. This shows that 48.6% of the students' scores are between 90 and 120. It also shows that the students' overall basic knowledge is still very good, and there is much room for improvement.

Table III-2. Descriptive statistics of mathematics scores of high school students.

	Cases	Minimum Value	Maximum Value	Average Value		Standard Deviation
	Statistics	Statistics	Statistics	Statistics	Standard Error	Statistics
Math scores	105	46	148	106.9905	2.18358	22.37508

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	Cases	Minimum Value	Maximum Value	Average Value		Standard Deviation
	Statistics	Statistics	Statistics	Statistics	Standard Error	Statistics
Number of valid cases (column)	105					

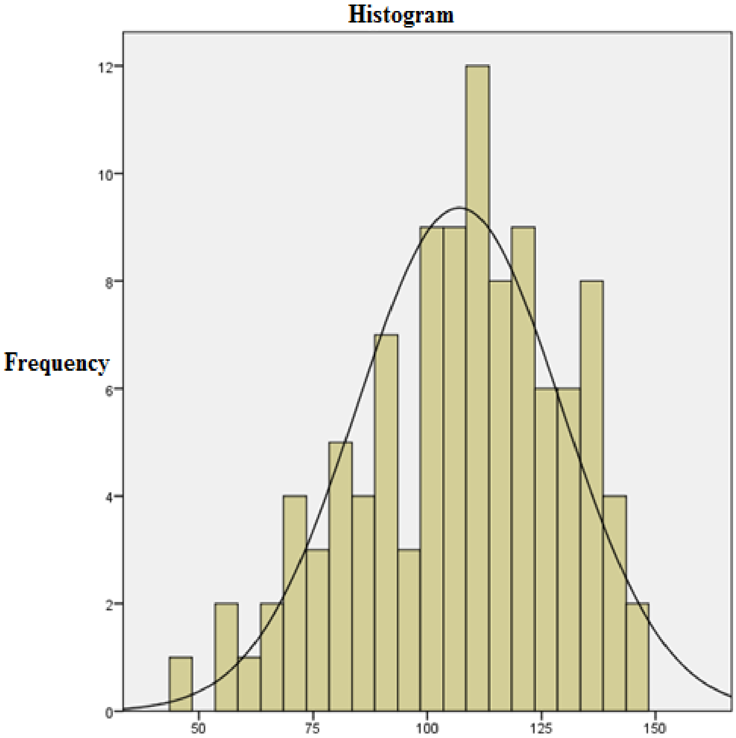

Fig. III-5. Statistics diagram of the survey of high school students' math scores.

Secondly, according to question 2 and questions 3 and 4, it was found that senior high school students encountered more difficulties in solving problems. Only 5 senior high school students said they would not encounter difficulties. The main reasons for encountering difficulties are the lack of problem-solving ideas, difficulty in grasping knowledge, and inadequate understanding of topics. 25% did not solve the problem, about 23% had difficulty grasping knowledge points, and the lack of inflexible skills accounted for most, about 31%.

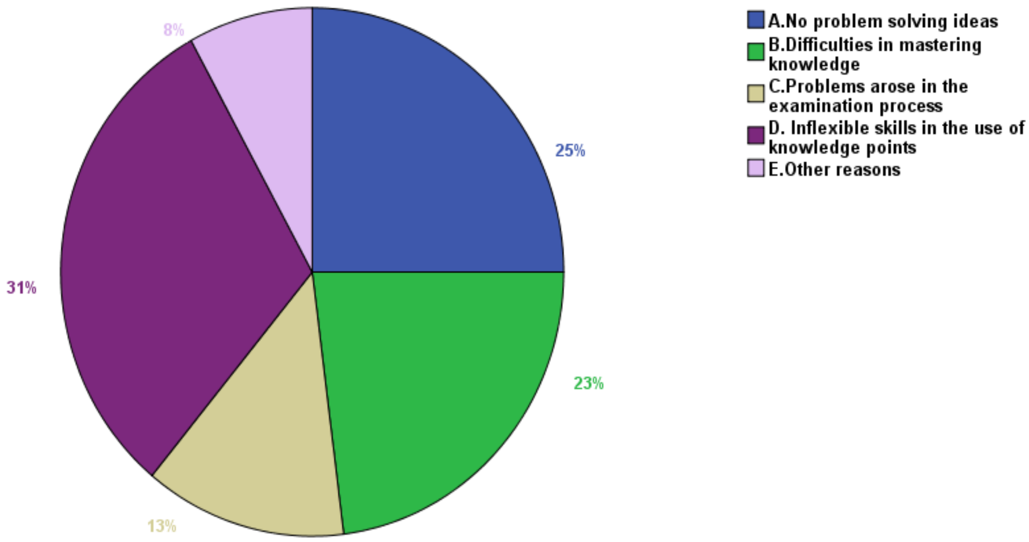

Fig. III-6. Statistics diagram of the reason for the difficulty of senior high school students in solving problems.

Questions 9, 12, and 13 mainly investigate the thinking and psychological situation of senior high school students when solving problems. Analyze whether the problem-solving failure is related to psychological factors. Seventy percent of students said that they had more or less bad psychology, such as lack of confidence and refusal to study mathematics. Students will feel that this problem cannot be solved and give up solving the problem without making actual calculations. It can be seen that the failure to solve problems is also related to the psychological factors of high school students.

The rest of the questions examine the problem solving habits of senior high school students from many aspects. Almost 19% of the students in the topic is not an error while 57% of high school students the topic not carefully would happen occasionally. Therefore, the formation of good examination habits also need to pay special attention. Question 15 also shows that most senior high school students do not have the ability to reflect on the problem. 39% of the students said that their problem solving reflection was only "fleeting" with the teacher’s explanation in the classroom. So in addition to individual students to develop good study habits, teachers also may carry on the effective organization through classroom teaching and to improve senior high school students ability to solve problems.

C. *The Causes of Failure of Senior High School Students in Solving Triangle Problems*

Through the above research, the reasons for the failure of senior high school students in solving triangle problems can be summarized as the following:

1. *Students have not Mastered the Basic Knowledge Required to Solve the Triangular Problem and do not Know the Steps Required to Solve the Problem.*

The knowledge required in the process of solving the triangle mainly includes the theorem of sine and cosine. The formula of the area of the triangle, and some characteristics of the triangle itself: if the sum of the three internal angles of the triangle is equal to 180 °, the sum of the two sides of the triangle is greater than the third side, and the difference between the two sides is less the third side. The sine and cosine expansion formula of the sum and difference of the two angles.

2. *It is Difficult to Apply the Learned Knowledge to Practical Problems without Truly Understanding the Learned Knowledge.*

For example, according to the known conditions in the title, the angle between the two sides and the other side of the triangle is known, and it is not known to use cosine to understand the triangle. Knowing the two angles of the triangle, it is unclear whether the two-angle sum and difference sine. Not sure cosine expansion formula and triangle induction formula should be used.

3. *The “No Idea” Caused by the Rejection of Problem Solving Caused by Personal Psychological Factors Caused the Failure to Understand the Problem.*

Students have abstract, complex and difficult ideas about mathematics itself. Seeing unfamiliar problems has caused fear and abandonment. They take it for granted that this problem cannot be solved and cause understanding problems to fail.

IV. High School Students' Failure to Solve the Triangle Problem

Polya analyzed the macro-thinking process of mathematical problem solving and believed that mathematical problem solving should be divided into four steps: understanding the problem, drawing up the plan, implementing the plan, reviewing and testing [8]. The same should be true for the triangle problem solving process. We must figure out the problem, find the known unknown, and draw the appropriate symbols into the drawing. Find out the connection between the known and the unknown. If there is no direct connection, you should consider the auxiliary problem. Finally came a solution plan. Implement the solution plan and verify each step.

A. *Master the Basic Knowledge and Necessary Derivation Process Required to Solve the Triangle*

The basic knowledge involved in solving the triangle part mainly includes: sine theorem, cosine theorem, and the conclusion that the triangle itself has: the sum of the three internal angles is equal to 180 °, the sum of the two sides is greater than the third side, and the difference between the two sides is only less than the third side. Induction formulas of trigonometric functions, trigonometric identities, basic formulas of plane vectors, sine and cosine expansion of two corner sums and differences. On the basis of understanding the essence of the concept, remember the concept with concise symbolic language or mantra. For example, the derivation of sine theorem, cosine theorem, and cosine theorem can use mathematical symbolic language instead of text language; the memory of the trigonometric function induction formula, the sine and cosine expansion of the two corners and the difference can be used. Cultivate good study habits, form a clear mathematical knowledge thinking system, and summarize the old and new concepts to form a reasonable and orderly concept system.

B. *Students Work Diligently to Cooperate with Teachers' Teaching*

Students should practice more on the basis of mastering the knowledge of solving triangles. Solving triangle problems can be divided into the problem of the number of triangles, the problem of finding the side length, the angle, the area, and the shape of the triangle. For different types of exercises, a general classification should be done, and the exercises should be sorted out one by one. Teachers should guide students to think in the teaching process, reasonably use teaching methods, teach the corresponding problem - solving methods, arrange reasonable feedback links, and check students' learning status in detail.

C. *Enhance Learning Confidence*

There are individual differences in students' ability to solve problems, which has a great influence on problem solving. For students with poor learning ability, while cultivating good learning habits, they should pay more attention to cultivate positive emotions, overcome the fear of unfamiliar difficult exercises, and increase interest in learning. A good mentality can help any stage of learning [9]. Students can exercise quality through personal efforts to actively develop good math learning habits. Teachers can infiltrate more mathematics culture about mathematics spirit in classroom teaching. Bringing students closer to mathematics makes students feel that mathematics is useful. They are not so far away. Faced with difficulties will not be discouraged.

V. Conclusion

Solving triangles is conducive to cultivating students' mathematical thinking methods that combine numbers and shapes. It can improve senior high school students' learning ability and thinking. In the college entrance examinations over the years, it also has a certain proportion, but many senior high school students are currently experiencing difficulties in learning and solving problems in solving triangles. The main reasons are difficulty in mastering knowledge, improper thinking and psychological fear. Students' personal efforts to coordinate with teachers' teaching can solve problems. Reasonable teaching methods help students to memorize knowledge and form good thinking methods. Reasonable adjustment of mentality and emotion helps students to establish a good learning concept. Develop good learning habits for high school students and improve problem-solving skills. It will also greatly promote future learning and development.

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